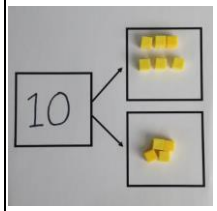
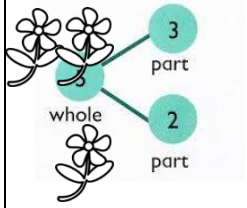
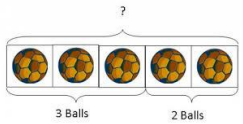
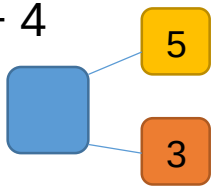
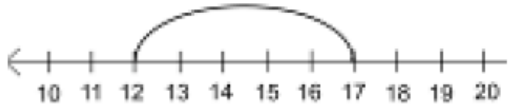
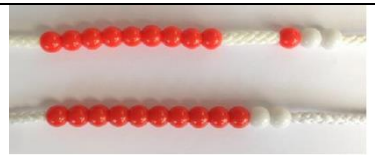
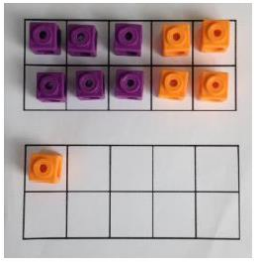
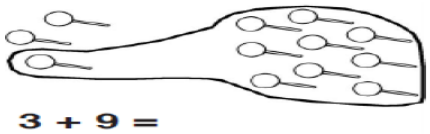
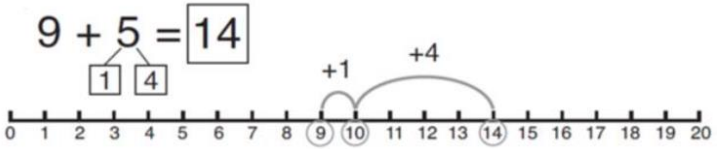
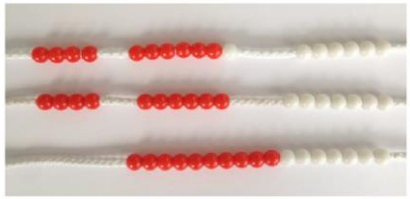
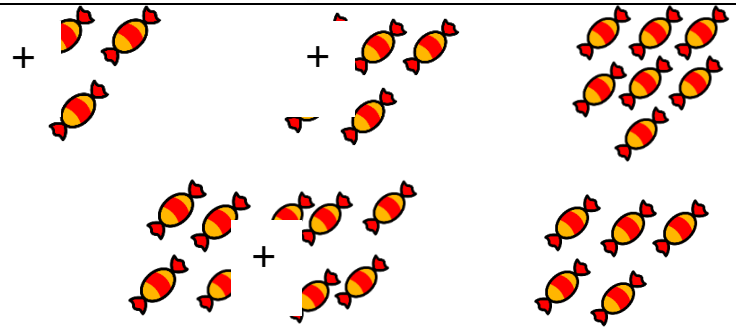
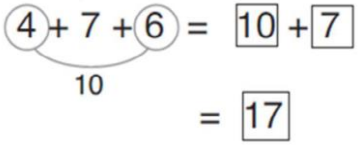
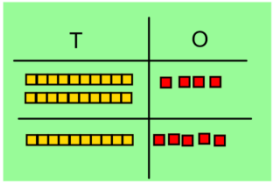
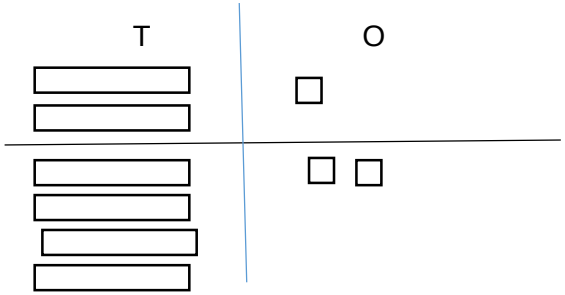
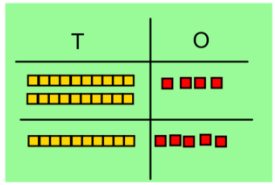
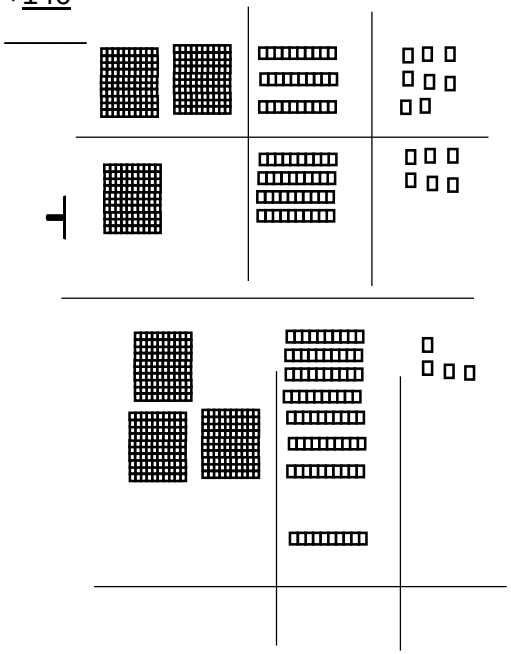
Egginton Primary School Progression in Calculations

Addition

Objective and Strategies	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part-whole model	  Use cubes to add two numbers together as a group or in a bar.   Also use Numicon resources for adding two parts to make a whole.	  Use pictures to add two numbers together as a group or in a bar. 	4 + 3 = 7 10= 6 + 4  Use the part-part whole diagram as shown above to move into the abstract.
Starting at the bigger number and counting on	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	12 + 5 = 17  Start at the larger number on the number line or number track and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the

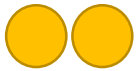
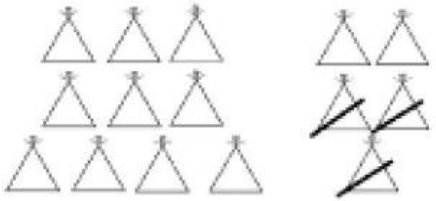
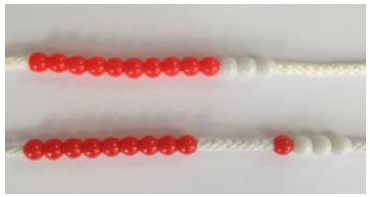
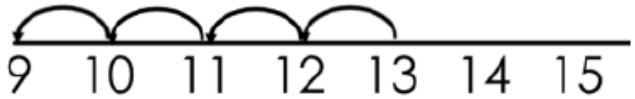
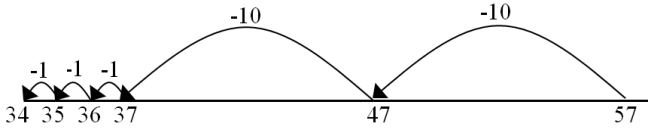
	Counting on using large and small number tracks.		smaller number to find your answer.
Regrouping to make 10.	 6 + 5 = 11  Start with the bigger number and use the smaller number to make 10.	 3 + 9 = Use pictures or a number line. Regroup or partition the smaller number to make 10.  	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now?
Adding three single digits	4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7.  Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	 Add together three groups of objects. Draw a picture to recombine the groups to make 10.	 Combine the two numbers that make 10 and then add on the remainder.

Column method- no regrouping	24 + 15= Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.  	After practically using the base 10 blocks and place value counters, children can draw the base 10 blocks to help them to solve additions.  	Calculations 21 + 42 = 21 + 42

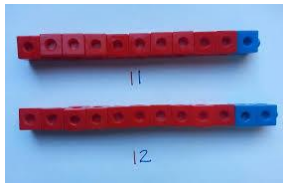
Column method- regrouping	Make both numbers on a place value grid.  Add up the units and exchange 10 ones for one 10. 238 +146  Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.	Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding.	Start by partitioning the numbers before moving on to clearly show the exchange below the addition. 20 + 5 40 + 8 60 + 13 = 73 As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here. 536 + 85 621 1 1 72.8 + 54.6 127.4 1 1 £ 2 3 . 5 9 + £ 7 . 5 5 £ 3 1 . 1 4 1 1 1 23.361 + 9.08 + 59.77 + 1.3 59.770 23.361 9.080 + 1.300 93.511 2 1 2 Note: Pupils need to include the place value holders (0)

	This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100. As children move on to decimals, money and decimal place value counters can be used to support learning.		to make each number in the calculation have the same number of decimal places.

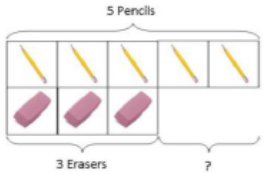
Subtraction

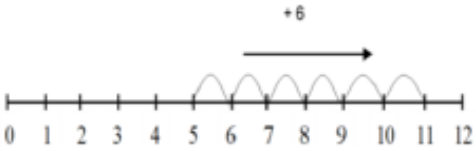
Objective and Strategies	Concrete	Pictorial	Abstract
Taking away ones	Use physical objects, counters, cubes, buckets and Numicon etc to show how objects can be taken away.  6 – 2 = 4   	Cross out drawn objects to show what has been taken away.  15 – 3 = 12	18 -3= 15 8 – 2 = 6
Counting back	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.  13 – 4 Use counters and move them away from the group as you take them away counting backwards as you go. 	Count back on a vertical and horizontal number line or number track  9 10 11 12 13 14 15 Start at the bigger number and count back the smaller number showing the jumps on the number line.  -10 -10 -1 -1 -1 34 35 36 37 47 57 This can progress all the way to counting back using two 2 digit numbers.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help.

<table>
<tr>
<td>Find the difference</td>
<td>Compare amounts and objects to find the difference.

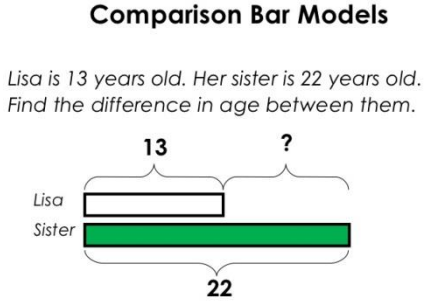
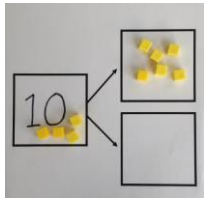
Use cubes to build towers or make bars to find the difference

Use basic bar models with items to find the difference</td>
<td> 

Count on to find the difference.

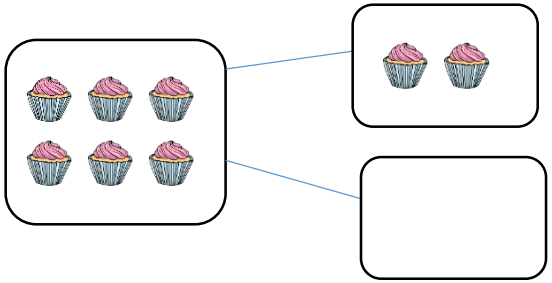
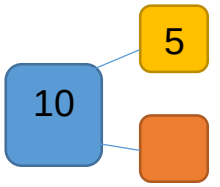
Draw bars to find the difference between 2 numbers.

 
</td>
<td>Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.</td>
</tr>
<tr>
<td>Part Part Whole Model</td>
<td> 

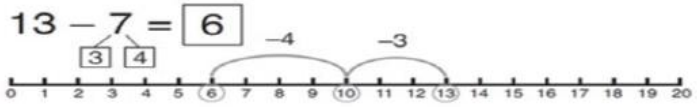
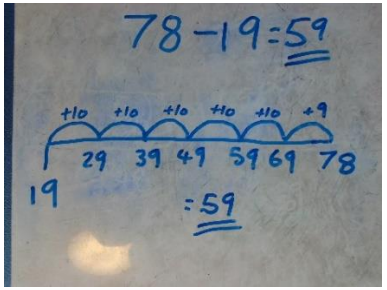
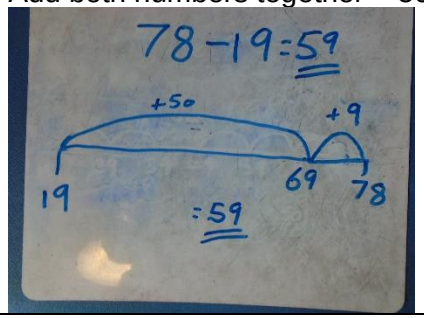
Link to addition- use the part whole model to help explain the inverse between addition and subtraction.

If 10 is the whole and 6 is one of the parts. What is the other part?

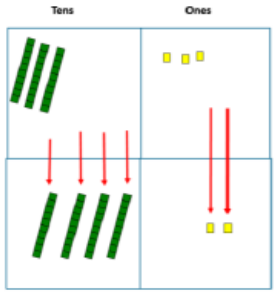
10 - 6 =</td>
<td>Use a pictorial representation of objects to show the part part whole model.

  </td>
<td> 

Move to using numbers within the part whole model.</td>
</tr>
</table>

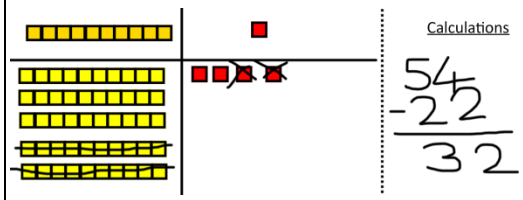
Make 10	14 – 9 =  Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.	 Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.	16 – 8= How many do we take off to reach the next 10? How many do we have left to take off?
Subtraction using a numberline.		78 – 19 = Start with the lowest number on the left of the line.  Count in tens until the nearest ten below the target number of 78. (E.G. 19, 29, 39, 49, 59, 69 = 5 tens = 50). Count in ones until you reach your target number (E.G. 6= 70, 71, 72, 73, 74, 75, 76, 77, 78 = 9). Add both numbers together = 50 + 9 = 59. Leading to:  	Leading to mental methods of using a numberline for subtraction of numbers found close together on the numberline. EG: 205 – 196 = 9

Column method without regrouping

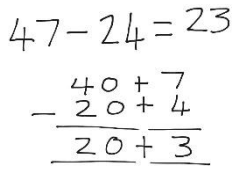


Use Base 10 to make the bigger number then take the smaller number away.

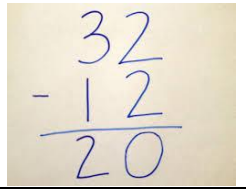
Show how you partition numbers to subtract. Again make the larger number first.



Draw the Base 10 or place value counters alongside the written calculation to help to show working.



This will lead to a clear written column subtraction.



Column method with regrouping

Use Base 10. Start with one exchange before moving onto subtractions with 2 exchanges.

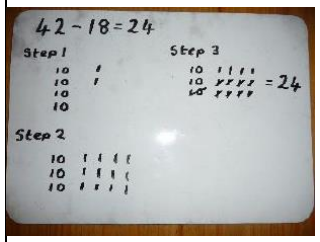
Make the larger number with the base 10.

234
- 88

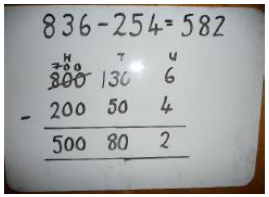
Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.

Draw the base 10 onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.

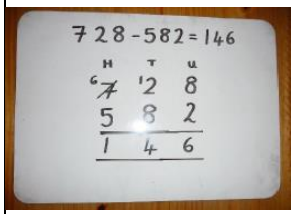
When confident, children can find their own way to record the exchange/regrouping.



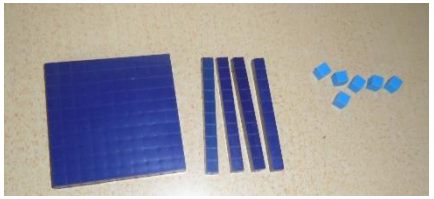
Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.



Children can start their formal written method by partitioning the number into clear place value columns.

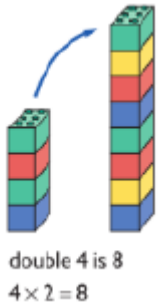
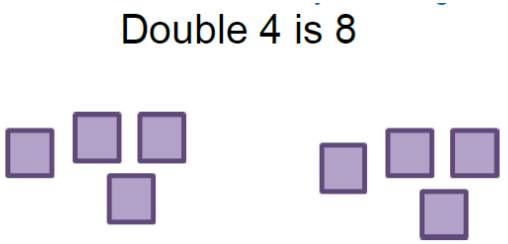
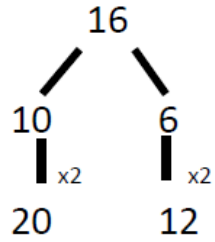
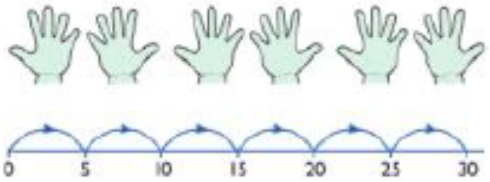


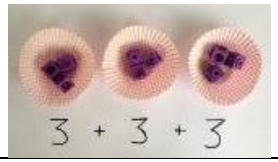
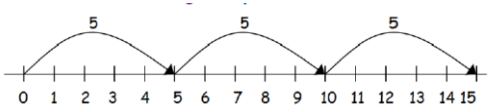
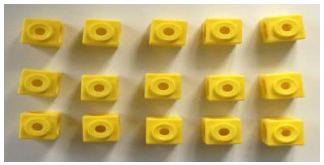
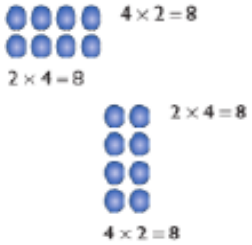
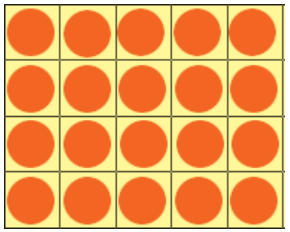
Moving forward the children use a more compact method.

	 Now I can subtract my ones. Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.  Now I can take away eight tens and complete my subtraction 		This will lead to an understanding of subtracting any number including decimals. E.G: 263 – 26.5 = $$\begin{array}{r} \overset{5}{\cancel{2}}\overset{12}{\cancel{6}}\overset{}{\cancel{3}}\overset{1}{.}\overset{}{\mathbf{0}} \\ -\quad 2\ 6\ .\ 5 \\ \hline 2\ 3\ 6\ .\ 5 \end{array}$$ Note: Pupils need to include the place value holders (0) to make each number in the calculation have the same number of decimal places.

	Calculations 1 2 3 4 - 8 8 1 4 6 Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.		

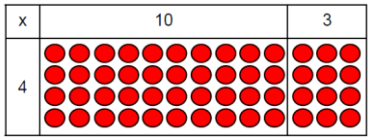
Multiplication

Objective and Strategies	Concrete	Pictorial	Abstract
Doubling	Use practical activities to show how to double a number.  Linked to symmetry. Doubling ladybird spots.	Draw pictures to show how to double a number.  	 Partition a number and then double each part before recombining it back together.
Counting in multiples	  Count in multiples supported by concrete objects in equal groups. Carry out songs and action songs. Also use Numicon tiles. Count in multiples on number tracsk and number lines.	 Use a number line or pictures to continue support in counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30

Repeated addition	  Use different objects to add equal groups. 	There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?  2 add 2 add 2 equals 6  5 + 5 + 5 = 15 What number sentence/problem does this calculation show?	Write addition sentences to describe objects and pictures. 
Arrays- showing commutative multiplication	Create arrays using counters/ cubes to show multiplication sentences.   How can we arrange these cubes in an array? Is there more than one way to do this?	Draw arrays in different rotations to find **commutative** multiplication sentences.   Link arrays to area of rectangles.	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

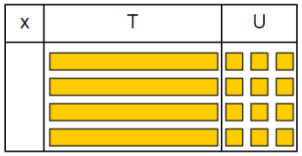
Grid Method

Show the link with arrays to first introduce the grid method.



4 rows of 10
4 rows of 3

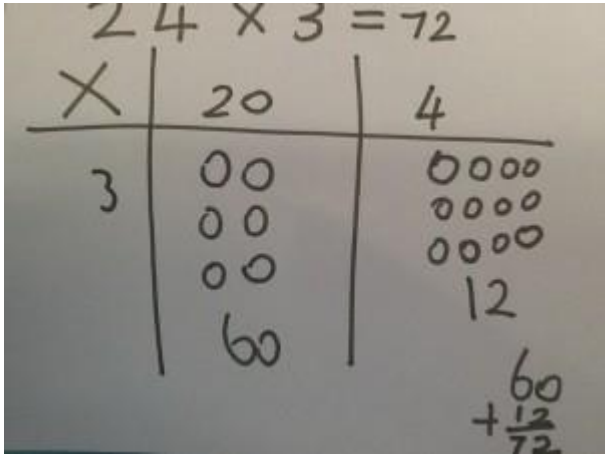
Move on to using Base 10 to move towards a more compact method.



4 rows of 13

Children can represent the work they have done with place value counters in a way that they understand.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.



Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

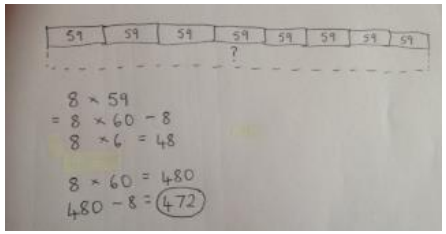
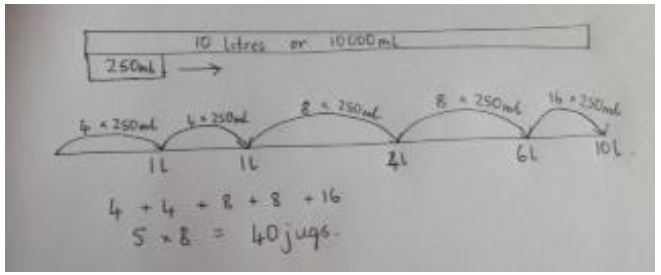
×	30	5
7	210	35

210 + 35 = 245

Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

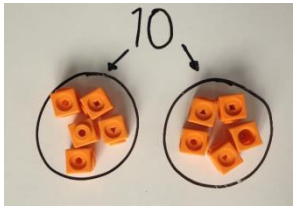
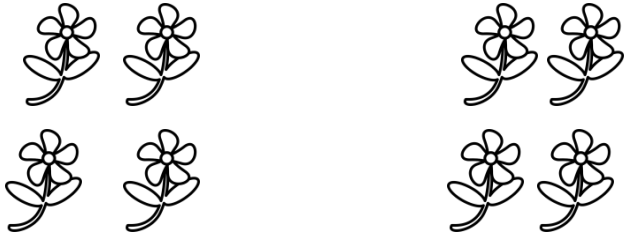
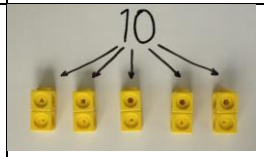
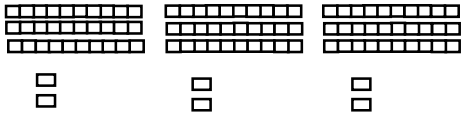
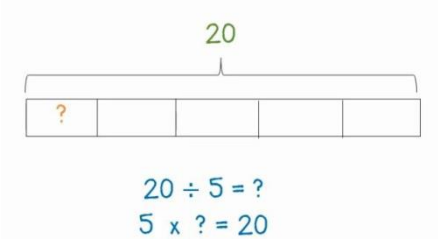
	10	8
10	100	80
3	30	24

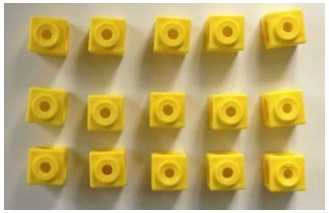
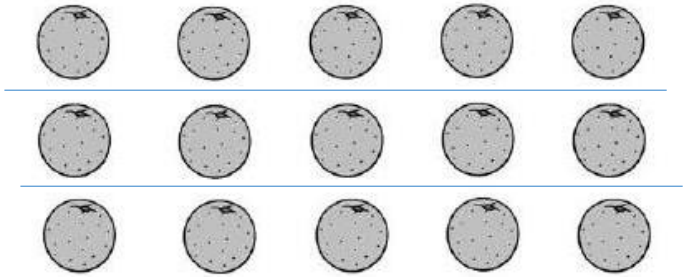
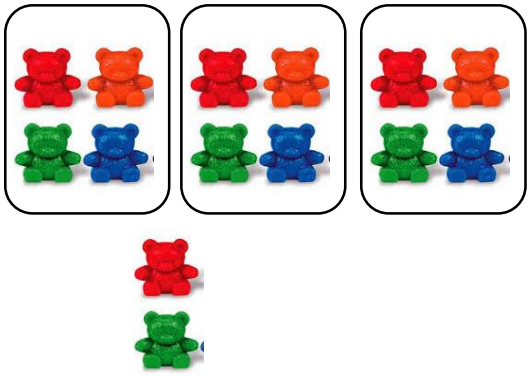
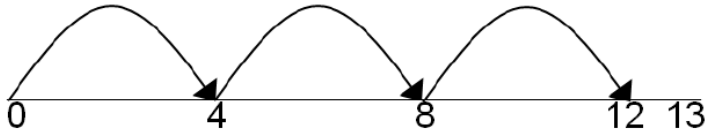
X	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

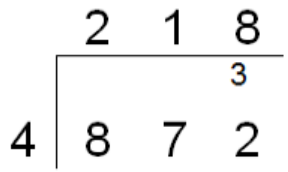
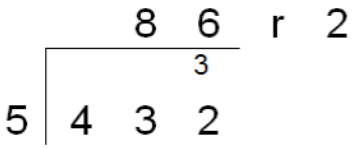
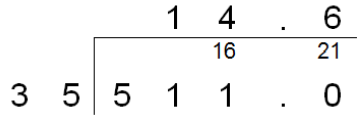
Column multiplication		(To be taught after abstract method in the right hand column). Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.   	1) TU x U E.G: 45 x 3 135 1 1 2) HTU x U and THTU x U E.G: 463 x 5 2315 2 3 1 3) TU.t x U and TU.th x U (including money and measures) E.G: £53.24 x 4. 00 £212.96 2 1 1 4) TU x TU (teens) Start with long multiplication, reminding the children about lining up their numbers clearly in columns. If it helps, children can write out what they are solving next to their answer. (See following page for details.)

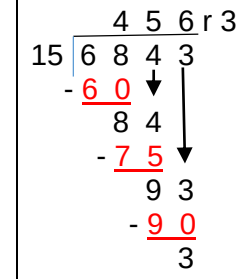
			32 x14 8 (4 x 2) 120 (4 x 30) 20 (10 x 2) + 300 (10 x 30) 448 This moves to the more compact method. 32 x14 128 (x4) +320 (x10) 448 5) TU x TU (E.G. 32 x 24) 6) HTU x TU 7) THTU x TU

Division

Objective and Strategies	Concrete	Pictorial	Abstract
Sharing objects into groups	 I have 10 cubes, can you share them equally in 2 groups? 	Children use pictures or shapes to share quantities.  8 ÷ 2 = 4	Share 9 buns between three people. 9 ÷ 3 = 3
Division as grouping	 Divide quantities into equal groups. Use cubes, counters or objects to aid understanding. 96 ÷ 3 = 32  12 ÷ 3 = 4 	Bar modelling (Teach AFTER abstract and concrete). Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. 	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?

Division within arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7
Division with a remainder	14 ÷ 3 = Divide objects between groups and see how much is left over (e.g. toys or pasta). 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder. 	Complete written divisions and show the remainder using r.  Note: Pupils need to learn the vocabulary ‘dividend’, ‘divisor’, ‘quotient’ and ‘remainder’ as they move through school.

Short division			Begin with divisions that divide equally with no remainder.  Move onto divisions with a remainder.  Convert the remainder to a fraction and then a decimal: E.G: 432 ÷ 5 = 86 r 2 = 86 2/5 = 86.4 Finally move into decimal places to divide the total accurately. 

Long division			Once pupils are confident with using short division, introduce long division for use with dividing larger numbers by two-digit numbers. E.G: 6843 ÷ 15 =  = 6843 ÷ 15 = 456 r3 = 456 3/15 = 456 1/5 = 456.2 (Numbers in red = 15 x 4 = 60 (nearest 'full 15' to 68); 15 x 5 = 75 (nearest 'full 15' to 84); 15 x 6 = 90 (nearest 'full 15' to 93). There is a remainder of 3 because we cannot get any 'full 15's' out of 3 therefore this becomes the remainder.